Solution theory for nonlinear partial differential delay equations

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Allen 411

Abstract. The object of study are evolutionary processes for which the time rate-of-change depends not only on the actual state but also on the history of the process. Typical examples are diffusive population models with temporal averages over the past, such as

\[
\begin{cases}
\frac{du}{dt}(t) - \Delta u(t) = au(t) \left[ 1 - bu(t) - \int_{-R}^{0} u(t + s) d\eta(s) \right], & t \geq 0 \\
u|_{[-R,0]} = \varphi
\end{cases}
\]

(production of red blood cells), as well as corresponding models with the Laplacian being replaced by more general, possibly nonlinear, diffusion/absorption operators.

In abstract form, such models lead to the following partial differential delay equations

\[
\begin{cases}
\dot{x}(t) + Bx(t) \ni F(x_t), & t \geq 0 \\
x|_I = \varphi \in \bar{E},
\end{cases}
\]

with \( B \subset X \times X \) a (generally) nonlinear and multivalued differential expression in a Banach space \( X \), and for given \( I = [-R,0], R > 0 \) (finite delay), or \( I = \mathbb{R}^- \) (infinite delay), and \( t \geq 0, x_t : I \to X \) the history of \( x \) up to \( t : x_t(s) = x(t + s), s \in I \).

The following basic problems will be addressed: existence, flow-invariance, and regularity of mild solutions.

There will be a reception for Dr. Reuss in Allen 467 at 3:30 pm.

Contact Michael Neumann, neumann@math.msstate.edu or (662) 325-7159, for additional information.